Lecture 33 - Review Exam 2

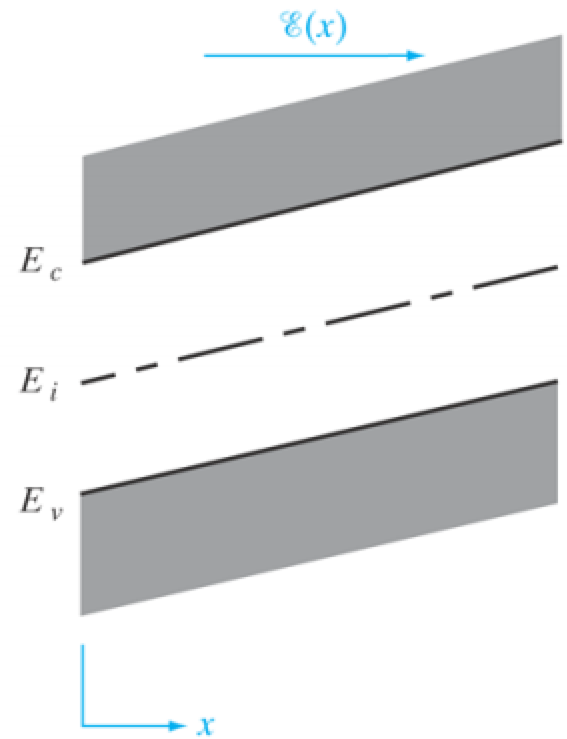
Spring 2022 – Section A
Prof. Ravaioli

Topics covered since Exam 1

- **Diffusion coefficient and diffusion length**
- ***p-n* junction**
- **Photodetectors**
- **LEDs and Lasers**
- **metal-semiconductor junction**

Einstein Relations

$$\mathcal{E}(x) = \frac{D_p}{\mu_p}\frac{1}{p(x)}p(x)\frac{1}{k_B T}\left[\underbrace{\frac{dE_i}{dx}}_{=q\mathcal{E}} - \underbrace{\frac{dE_F}{dx}}_{=0}\right]$$



$$\mathcal{E} = -\frac{dV}{dx} = -\frac{d}{dx}\left[\frac{E_i}{-q}\right] = \frac{1}{q}\frac{dE_i}{dx}$$

$$D_p = \frac{k_B T}{q}\mu_p$$

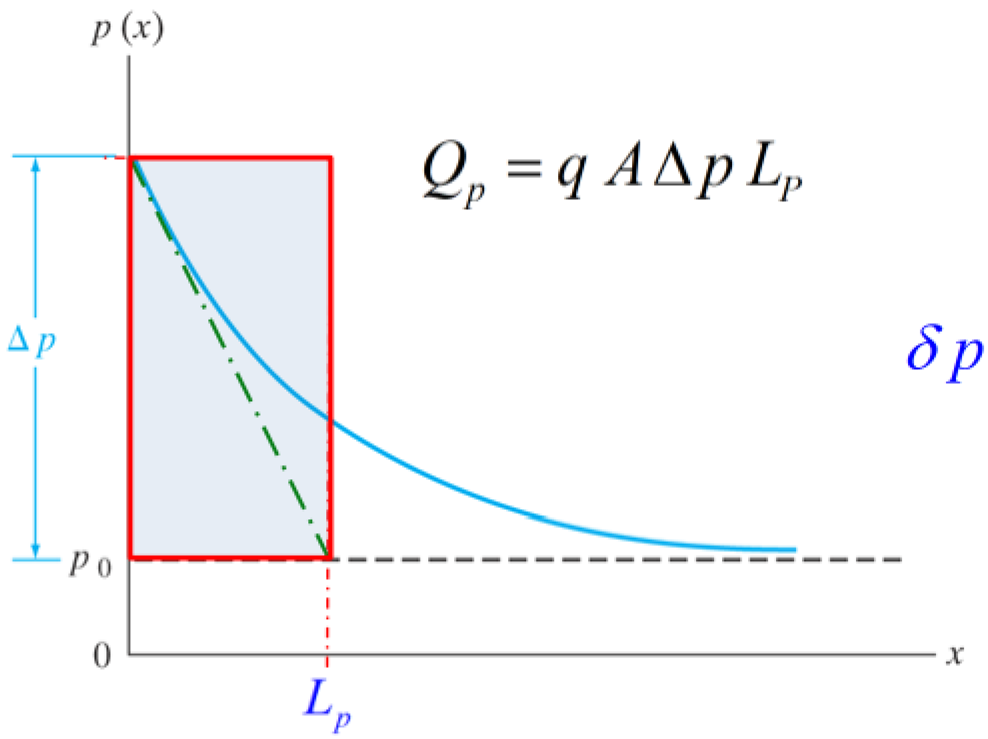
Einstein relation

Analogous result for electrons

$$D_n = \frac{k_B T}{q}\mu_n$$

Einstein relation

Diffusion Length



$$Q_p = q\,A\,\Delta p\,L_p$$

$$\delta p(x) = \Delta p \exp\left(-x/L_p\right)$$

Diffusion Length

Electrons $\sqrt{D_n \tau_n} = L_n =$ electron diffusion length

$$\frac{d^2}{dx^2}\delta n(x) = \frac{\delta n}{L_n^2}$$

Holes $\sqrt{D_p \tau_p} = L_p =$ hole diffusion length

p-n junction

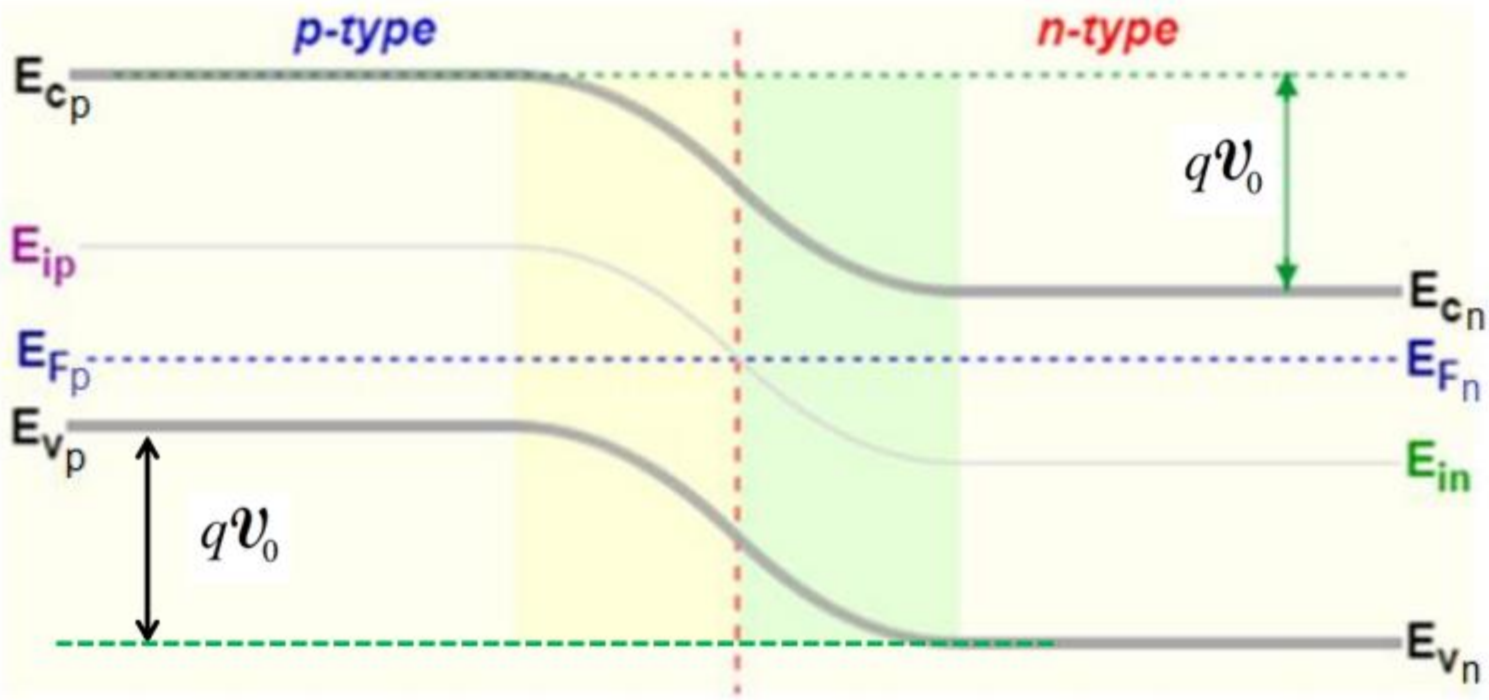
(equilibrium)

$$q\mathcal{V}_0 = E_{V_p} - E_{V_n}$$

$$q\mathcal{V}_0 = E_{C_p} - E_{C_n}$$

$$q\mathcal{V}_0 = E_{i_p} - E_{i_n}$$

$$E_{Fp} = E_{Fn}$$

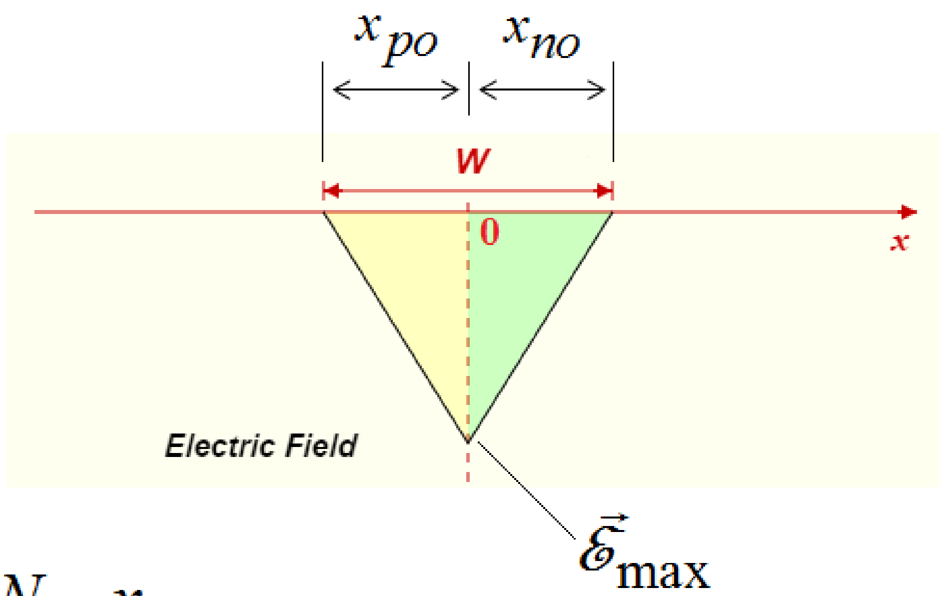


Depletion near the junction

The built-in potential is also related to the width of the depletion region. Since the field distribution has a triangular shape, the integral is simply

$$V_o = -\int_{-x_{po}}^{x_{no}} \vec{\mathscr{E}}(x)\,dx$$

$$= -\frac{1}{2}\vec{\mathscr{E}}_{\max} W$$



From Gauss Law

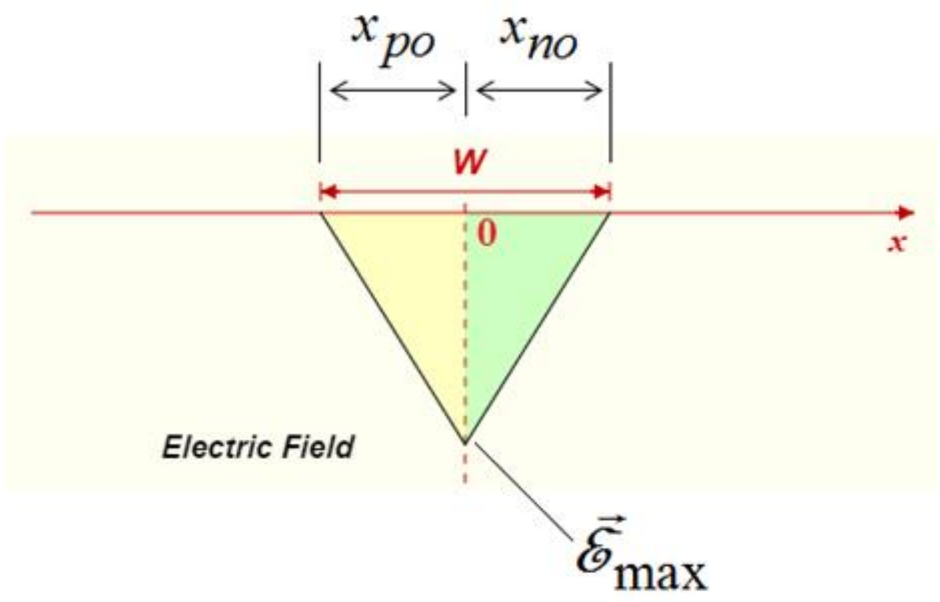
$$\left|\vec{\mathscr{E}}_{\max}\right| = q\frac{N_A\,x_{po}}{\varepsilon} = q\frac{N_D\,x_{no}}{\varepsilon}$$

Application of Gauss law

$$\int_0^{\mathscr{E}_0} d\mathscr{E} = -\frac{q}{\varepsilon} N_A \int_{-x_{p0}}^{0} dx \qquad [-x_{p0} < x < 0]$$

$$\int_{\mathscr{E}_0}^{0} d\mathscr{E} = \frac{q}{\varepsilon} N_D \int_0^{x_{n0}} dx \qquad [0 < x < x_{n0}]$$

$$\mathscr{E}_0 = \mathscr{E}_{\max} = -\frac{q}{\varepsilon} N_D\, x_{n0} = -\frac{q}{\varepsilon} N_A\, x_{p0}$$

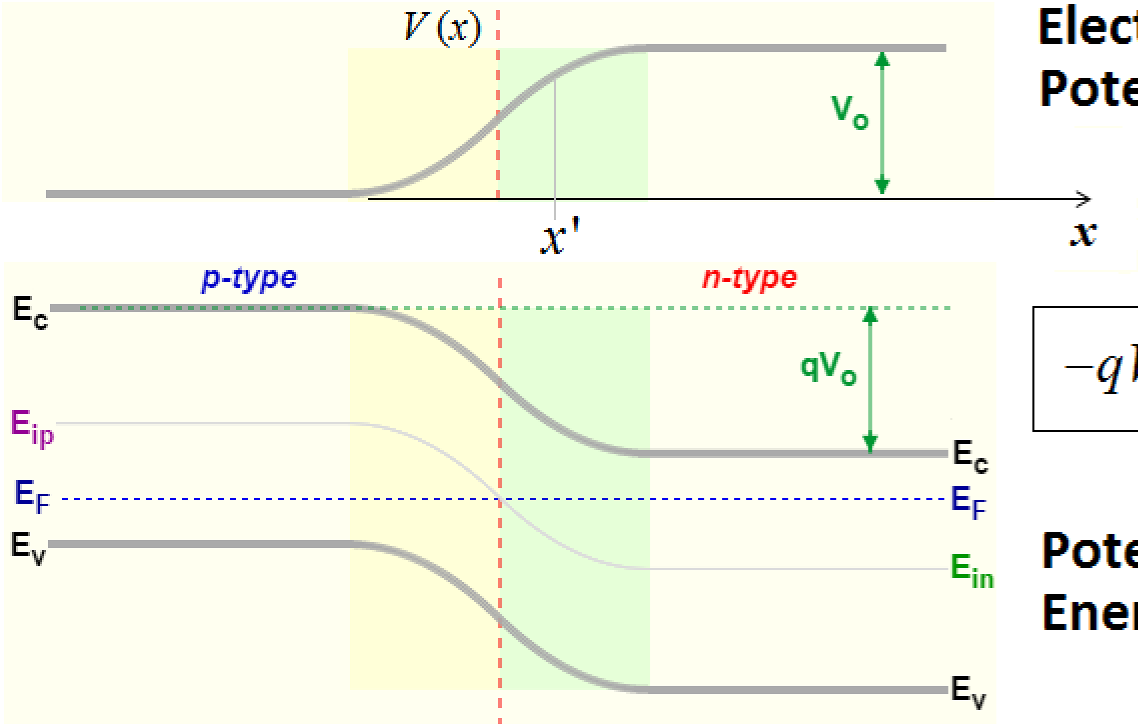


Details of band bending from potential

$V(x)$ **completes the band diagram in the depletion region**

$$-\frac{dV}{dx} = \vec{\mathcal{E}}(x)$$

$$-\int_{-\infty}^{x'} \vec{\mathcal{E}}(x)\, dx = V(x')$$



Electrostatic Potential

$-qV(x) = \mathbf{E}$

Potential Energy

p-n junction

(equilibrium)

Step junction: N_A on p-side & N_D on n-side

$$V_0 = \frac{k_B T}{q} \ln \frac{p_p}{p_n} = \frac{k_B T}{q} \ln \frac{N_A N_D}{n_i^2}$$

$$p_p \approx N_A \qquad p_n \approx \frac{n_i^2}{N_D}$$

Also:

$$\frac{p_p}{p_n} = \exp\left(\frac{qV_0}{k_B T}\right)$$

$$\frac{n_n}{n_p} = \exp\left(\frac{qV_0}{k_B T}\right)$$

In the regions far away from the junction

$$n_p = \frac{n_i^2}{p_p} \qquad n_n = \frac{n_i^2}{p_n}$$

p-n junction

(equilibrium)

Step junction: N_A on p-side & N_D on n-side

$$V_0 = \frac{k_B T}{q} \ln \frac{p_p}{p_n} = \frac{k_B T}{q} \ln \frac{N_A N_D}{n_i^2}$$

$$p_p \approx N_A$$

$$p_n \approx \frac{n_i^2}{N_D}$$

Also:

$$\frac{p_p}{p_n} = \exp\left(\frac{qV_0}{k_B T}\right)$$

$$\frac{n_n}{n_p} = \exp\left(\frac{qV_0}{k_B T}\right)$$

In the regions far away from the junction

$$n_p = \frac{n_i^2}{p_p}$$

$$n_n = \frac{n_i^2}{p_n}$$

p-n junction

$N_A = 10^{18}\text{cm}^{-3}$
$N_D = 5 \times 10^{15}\text{cm}^{-3}$

(equilibrium)

$$E_{ip} - E_F = k_B T \ln \frac{p_p}{n_i} = 0.0259 \ln \frac{10^{18}}{1.5 \times 10^{10}} = 0.467\,\text{eV}$$

$$E_F - E_{in} = k_B T \ln \frac{n_n}{n_i} = 0.0259 \ln \frac{5 \times 10^{15}}{1.5 \times 10^{10}} = 0.329\,\text{eV}$$

$$q\mathcal{V}_0 = E_{ip} - E_{in} = 0.467eV + 0.329eV = 0.796\,\text{eV}$$

$$q\mathcal{V}_0 = k_B T \ln \frac{N_A N_D}{n_i^2} = 0.0259 \ln \frac{5 \times 10^{33}}{2.25 \times 10^{20}} = 0.796\,\text{eV}$$

p-n junction

(equilibrium)

$$\Rightarrow \boxed{W = \sqrt{\frac{2\varepsilon V_o}{q}\frac{N_A + N_D}{N_A N_D}}}$$

Depletion Width

$N_A = 10^{18}\text{cm}^{-3}$
$N_D = 5 \times 10^{15}\text{cm}^{-3}$

$$qv_0 = k_B T \ln\frac{N_A N_D}{n_i^2} = 0.796\text{eV}$$

$$W = \sqrt{\frac{2\varepsilon V_0}{q}\frac{N_A + N_D}{N_A N_D}} = \sqrt{\frac{2\times\left(11.8\times 8.85\times 10^{-14}\right)\times 0.796}{1.6\times 10^{-19}}\cdot\frac{10^{18}+5\times 10^{15}}{10^{18}\times 5\times 10^{15}}}$$

$= 0.457\mu\text{m}$

p-n junction

(equilibrium)

$$\Rightarrow \boxed{W = \sqrt{\frac{2\varepsilon V_o}{q}\frac{N_A + N_D}{N_A N_D}}}$$

Depletion Width

$$N_A = 10^{18}\,\mathrm{cm}^{-3}$$
$$N_D = 5\times10^{15}\,\mathrm{cm}^{-3}$$

$$qv_0 = k_B T \ln\frac{N_A N_D}{n_i^2} = 0.796\,\mathrm{eV}$$

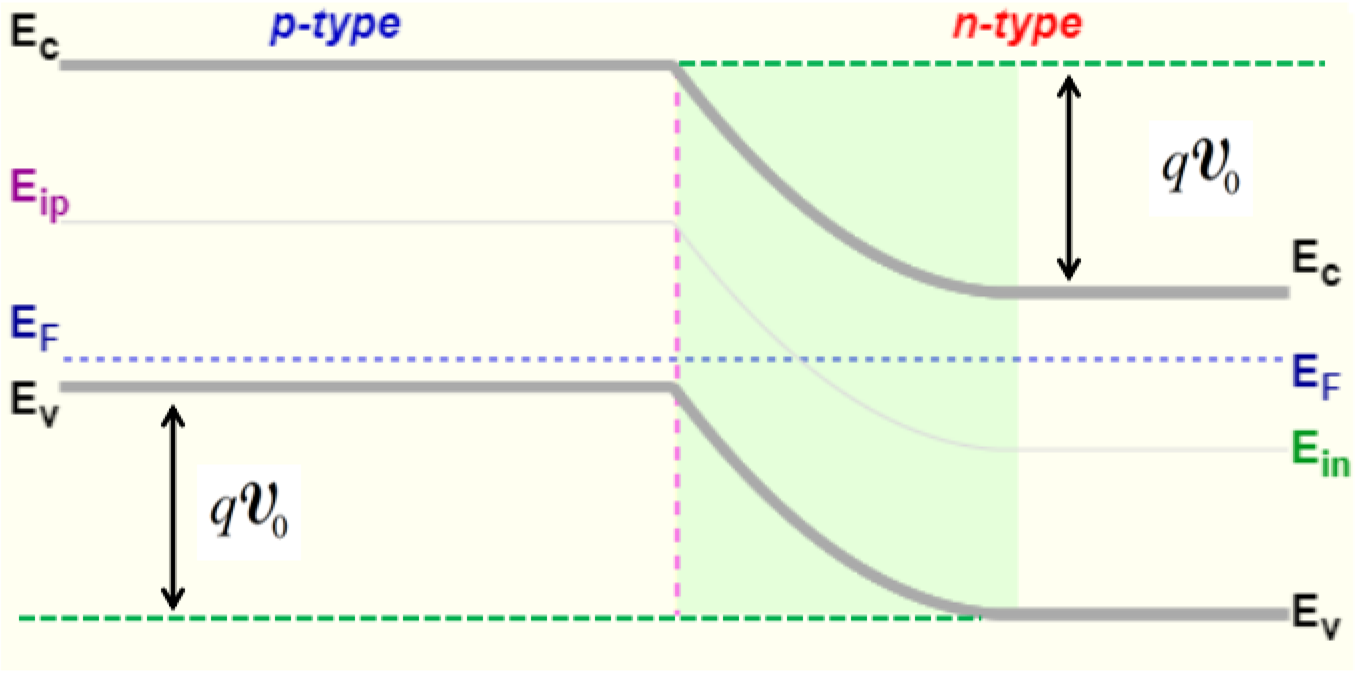
$$x_{po} = \frac{N_D}{N_A + N_D} W = \frac{5\times10^{15}}{10^{18} + 5\times10^{15}} 0.457 = 2.27\times10^{-3}\,\mu\mathrm{m}$$

$$x_{no} = \frac{N_A}{N_A + N_D} W = \frac{10^{18}}{10^{18} + 5\times10^{15}} 0.457 = 0.455\,\mu\mathrm{m}$$

p-n junction

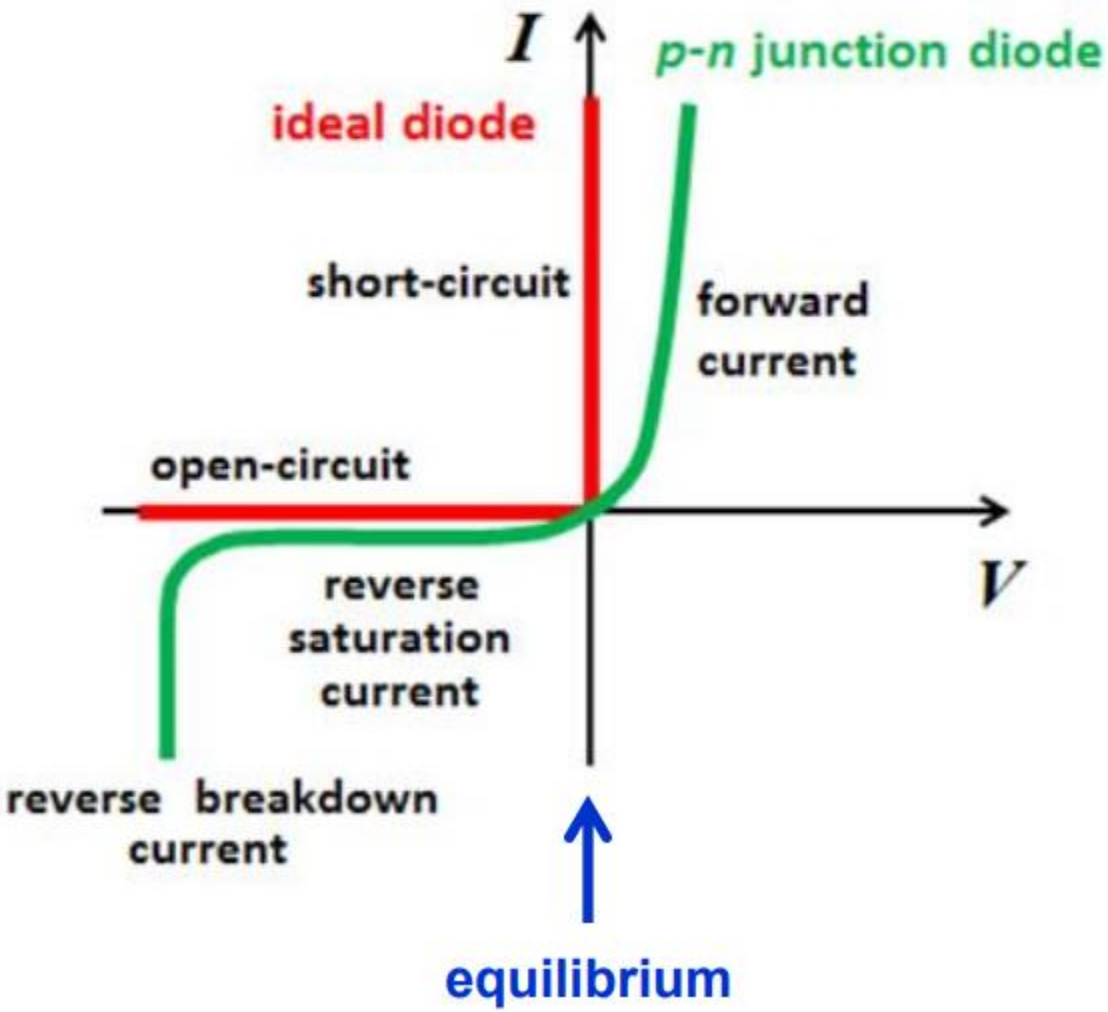
(equilibrium)

$N_A = 10^{18}\text{cm}^{-3}$
$N_D = 5 \times 10^{15}\text{cm}^{-3}$



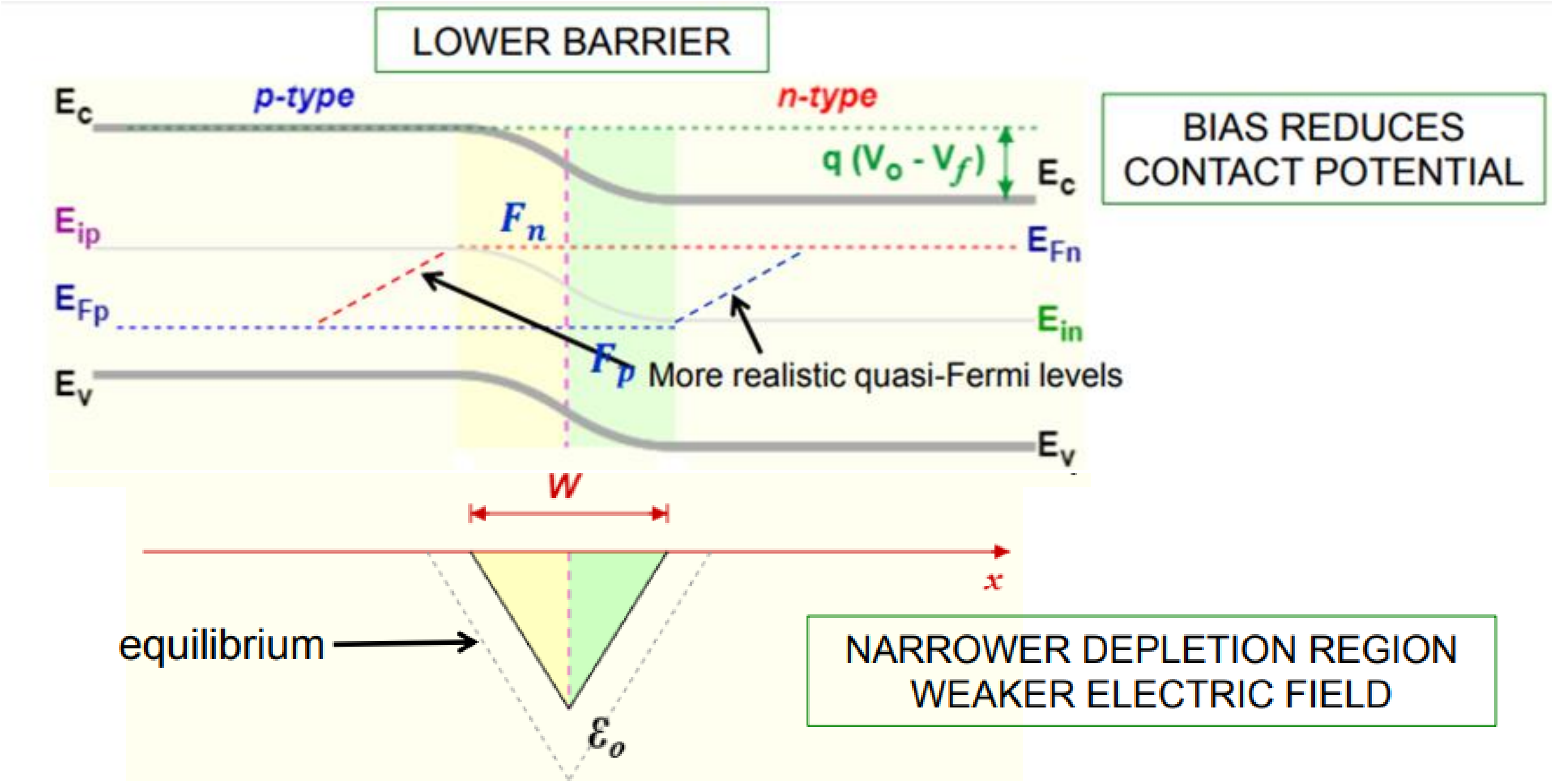
p-n junction

(out of equilibrium)



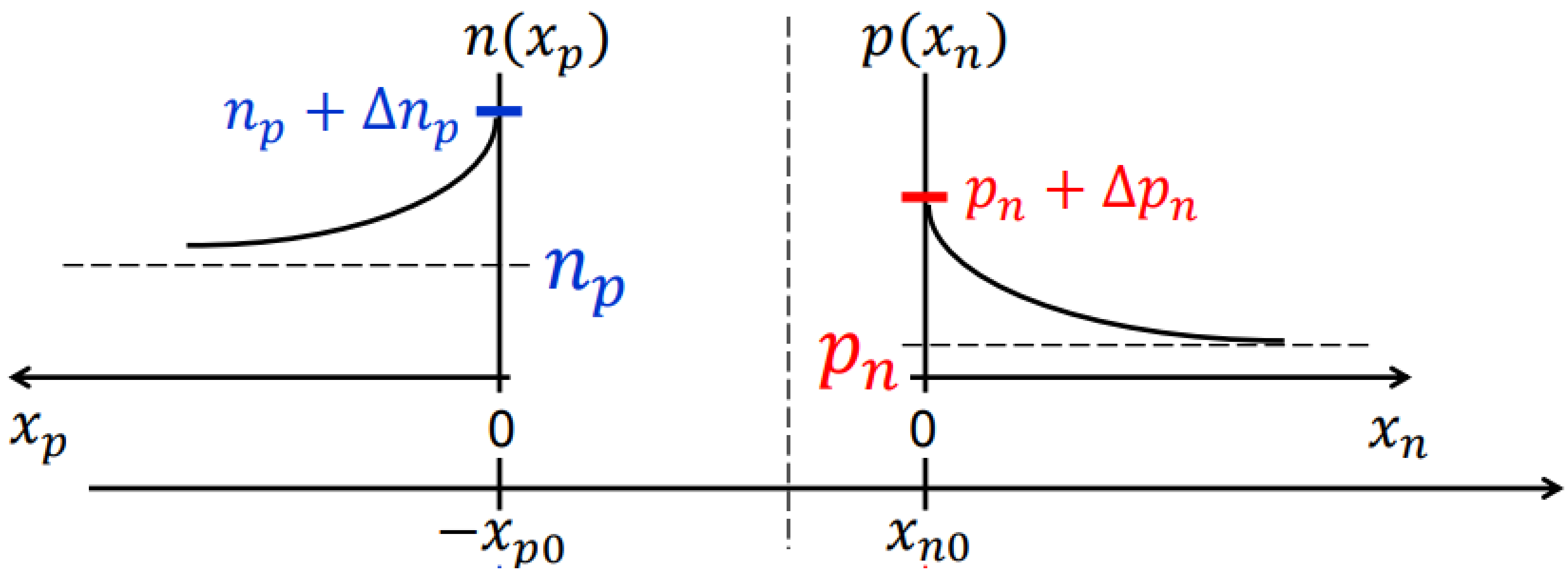
p-n junction

Forward bias



p-n junction

Forward bias



$$\Delta p_n = p_n \left[\exp\left(\frac{qV}{k_B T} \right) - 1 \right]$$

p-n junction

Forward bias

$$I = I_p(x_n = 0) - I_n(x_p = 0)$$

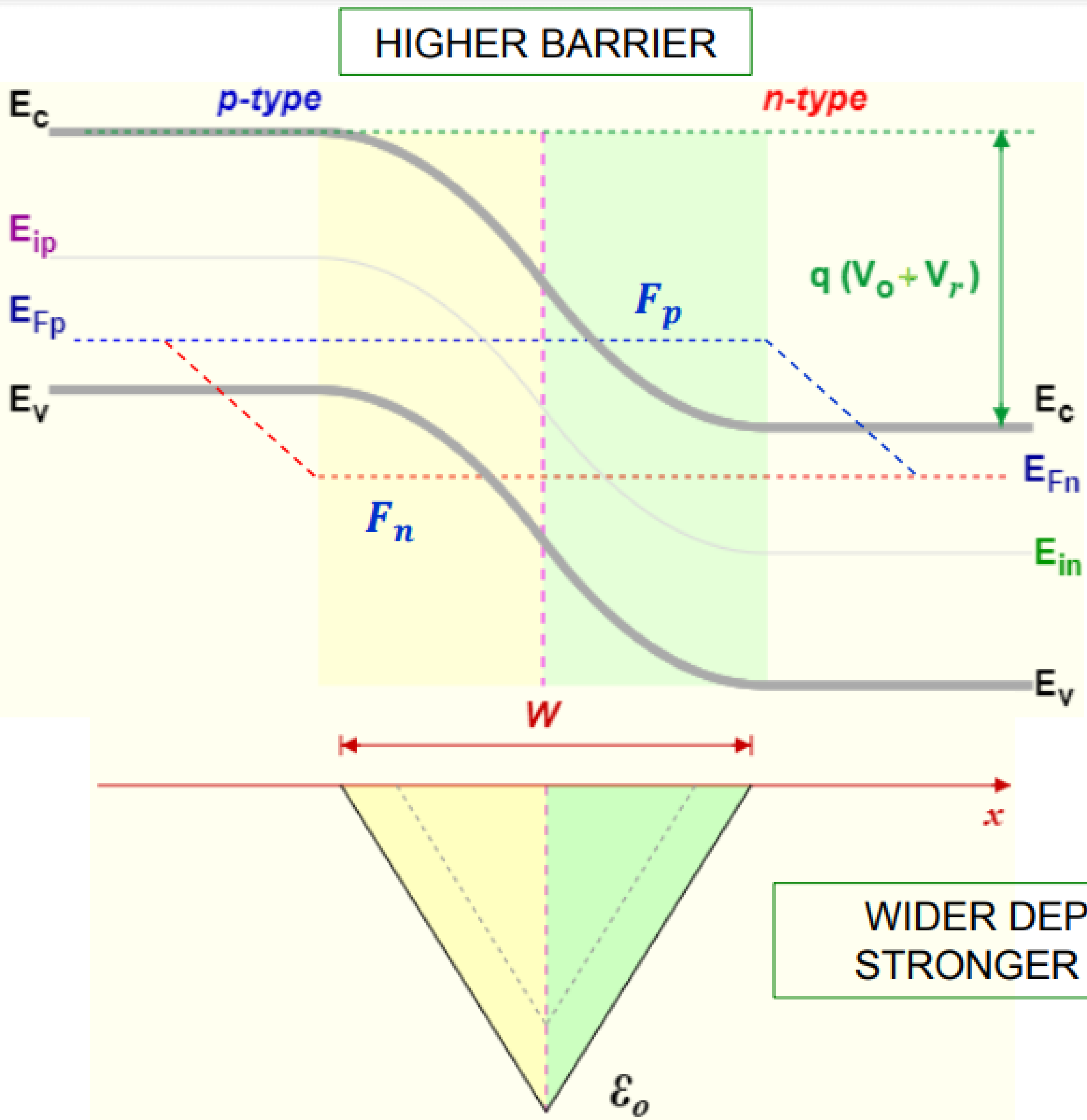
$$= qA\frac{D_p}{L_p}p_n\left[\exp\left(\frac{qV}{k_BT}\right) - 1\right] + qA\frac{D_n}{L_n}n_p\left[\exp\left(\frac{qV}{k_BT}\right) - 1\right]$$

$$= \underbrace{qA\left(\frac{D_p}{L_p}p_n + \frac{D_n}{L_n}n_p\right)}_{I_0}\left[\exp\left(\frac{qV}{k_BT}\right) - 1\right]$$

$$I = I_0\left[\exp\left(\frac{qV}{k_BT}\right) - 1\right]$$

p-n junction

Reverse bias



BIAS INCREASES CONTACT POTENTIAL

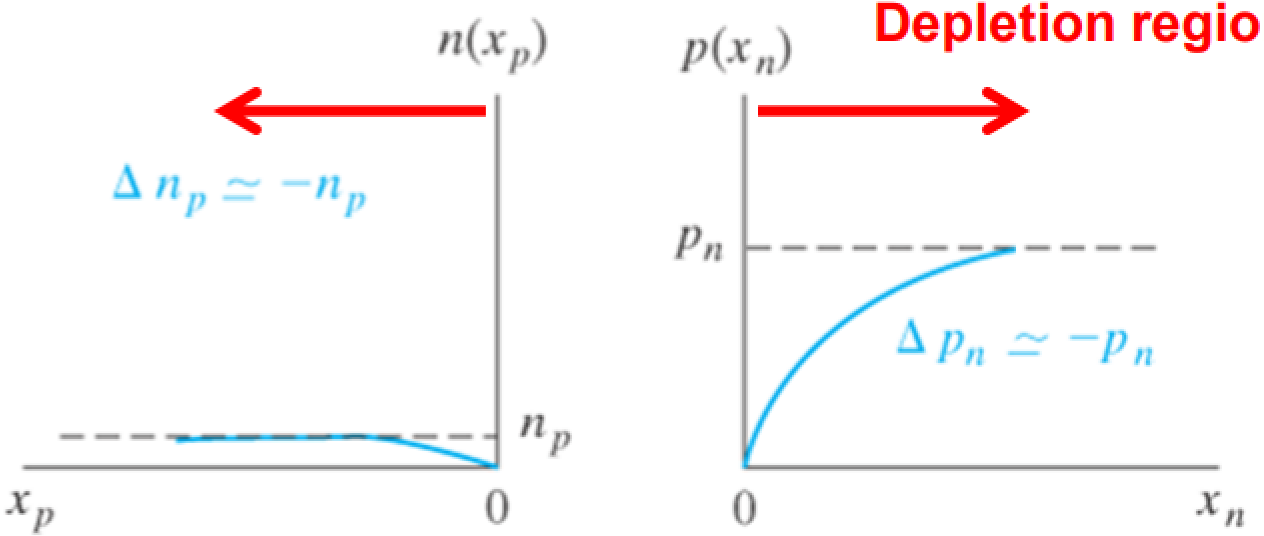
WIDER DEPLETION REGION
STRONGER ELECTRIC FIELD

p-n junction – from Lectures 24-25

Reverse bias

$V = -V_r$

Depletion region widens



p-*n* junction

Reverse bias

Reverse bias $V = -V_r$ with $V_r \gg k_B T / q$

$$\Delta p_n = p_n \left[\exp\left(-\frac{qV_r}{k_B T} \right) - 1 \right] \approx -p_n$$

$$\Delta n_p = n_p \left[\exp\left(-\frac{qV_r}{k_B T} \right) - 1 \right] \approx -n_p$$

p-n junction

Reverse bias

Reverse bias $V = -V_r$

$$I = I_0 \left[\exp\left(-\frac{qV_r}{k_B T} \right) - 1 \right]$$

$$I = -I_0 = -qA\left(\frac{D_p}{L_p} p_n + \frac{D_n}{L_n} n_p \right)$$

p-n junction

$$W = \sqrt{\frac{2\varepsilon V_0}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)}$$

Equilibrium

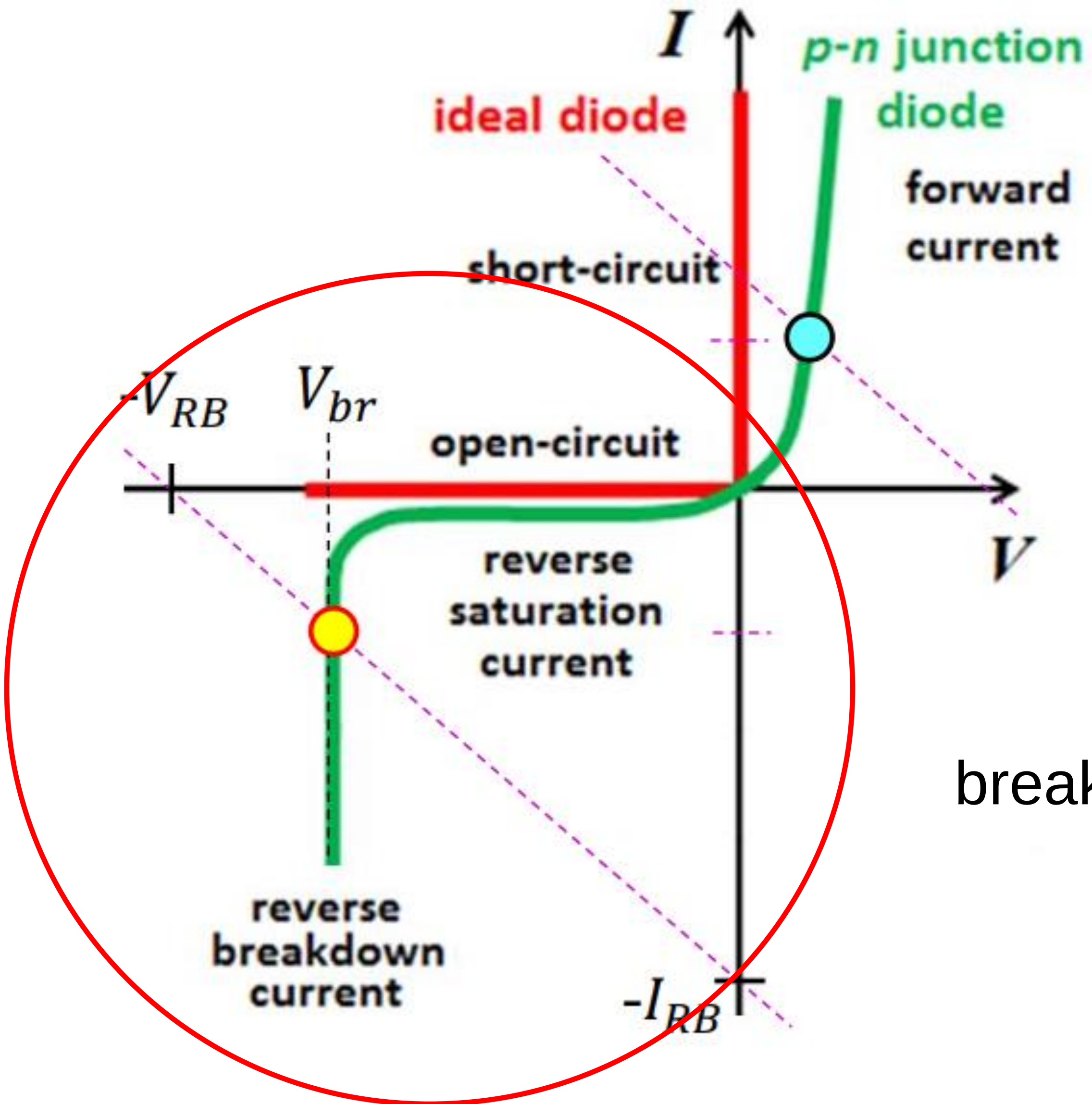
$$W = \sqrt{\frac{2\varepsilon (V_0 - V_f)}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)}$$

Forward Bias

$$W = \sqrt{\frac{2\varepsilon (V_0 + V_r)}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)}$$

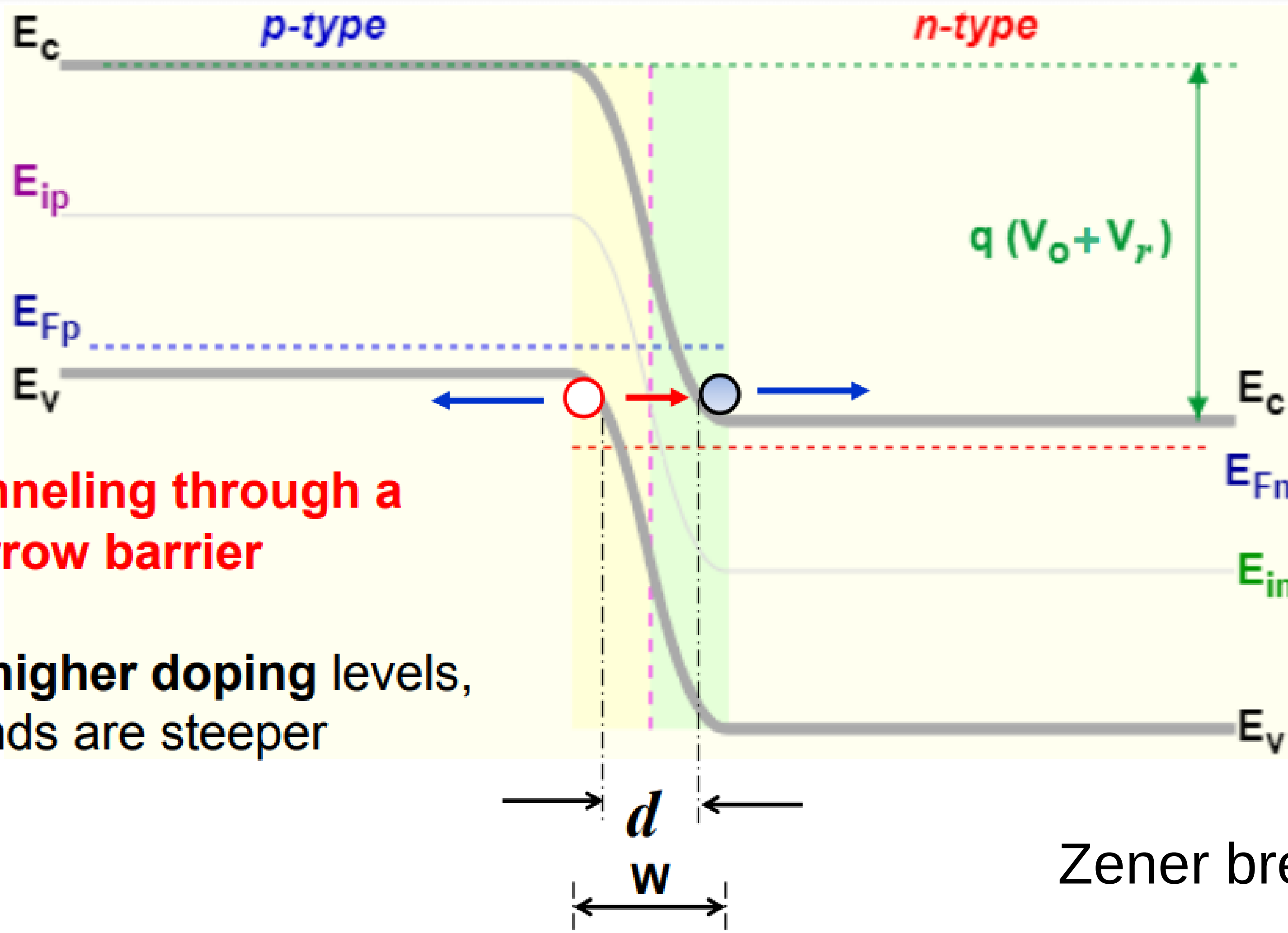
Reverse Bias

p-n junction –



breakdown

p-n junction



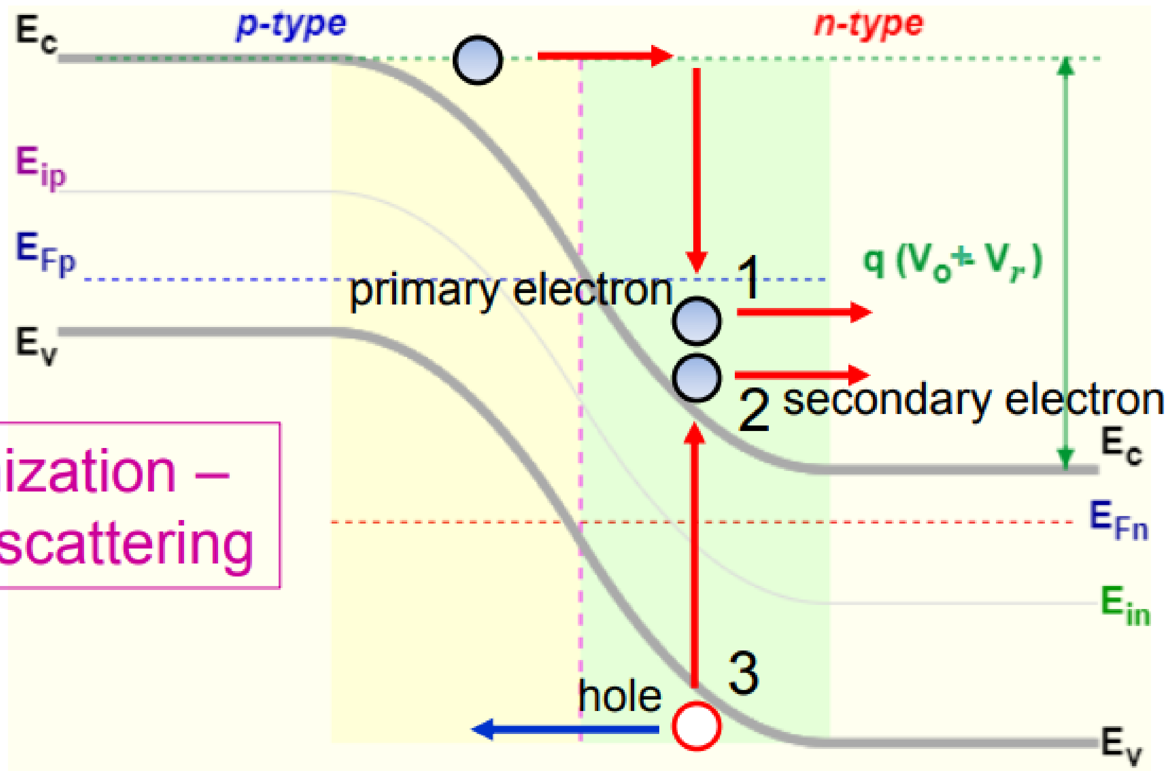
Tunneling through a narrow barrier

At **higher doping** levels, bands are steeper

Zener breakdown

p-n junction

Avalanche breakdown



impact ionization – 3-particle scattering

At relatively higher reverse voltages [about $q(V_0 - V_r) \gg E_g$], avalanche generation dominates, due to a high energy scattering mechanism called "**impact ionization**".

p-n junction

Junction Capacitance

$$|Q| = qAx_{n0}N_D = qAx_{p0}N_A$$

$$x_{n0} = \frac{N_A}{N_A + N_D}W \qquad x_{p0} = \frac{N_D}{N_A + N_D}W$$

$$|Q| = qA\frac{N_D N_A}{N_A + N_D}W$$

p-n junction

Junction Capacitance

$$|Q| = qA \frac{N_D N_A}{N_A + N_D} W = $$

$$= \epsilon A \sqrt{\frac{2q}{\epsilon}(V_0 - V)\frac{N_D N_A}{N_A + N_D}}$$

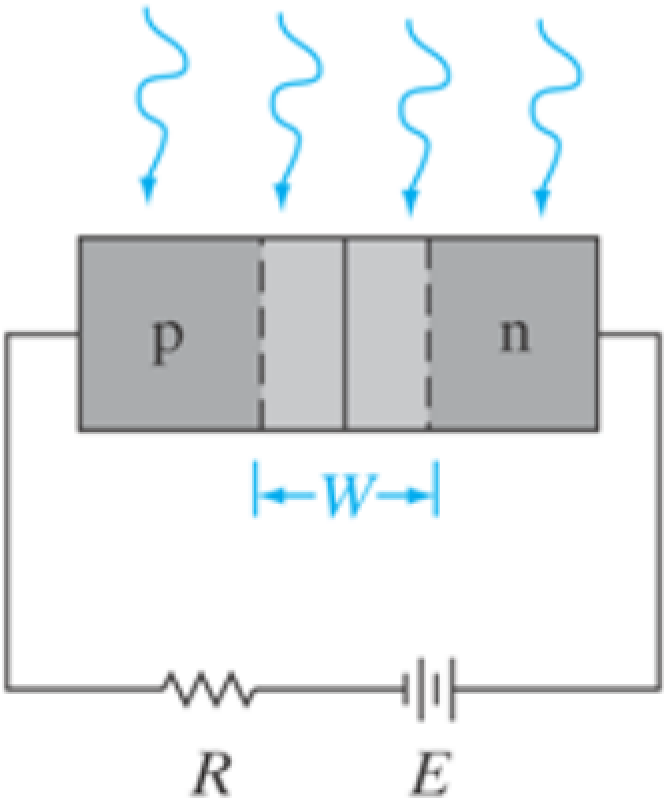
$$C_j = \left|\frac{dQ}{d(V_0 - V)}\right| = \epsilon A \underbrace{\sqrt{\frac{q}{2\epsilon(V_0 - V)}\frac{N_D N_A}{N_A + N_D}}}_{W^{-1}}$$

Optoelectronic devices

Photodiodes

- **Illuminated junction**

$$h\nu > E_g$$



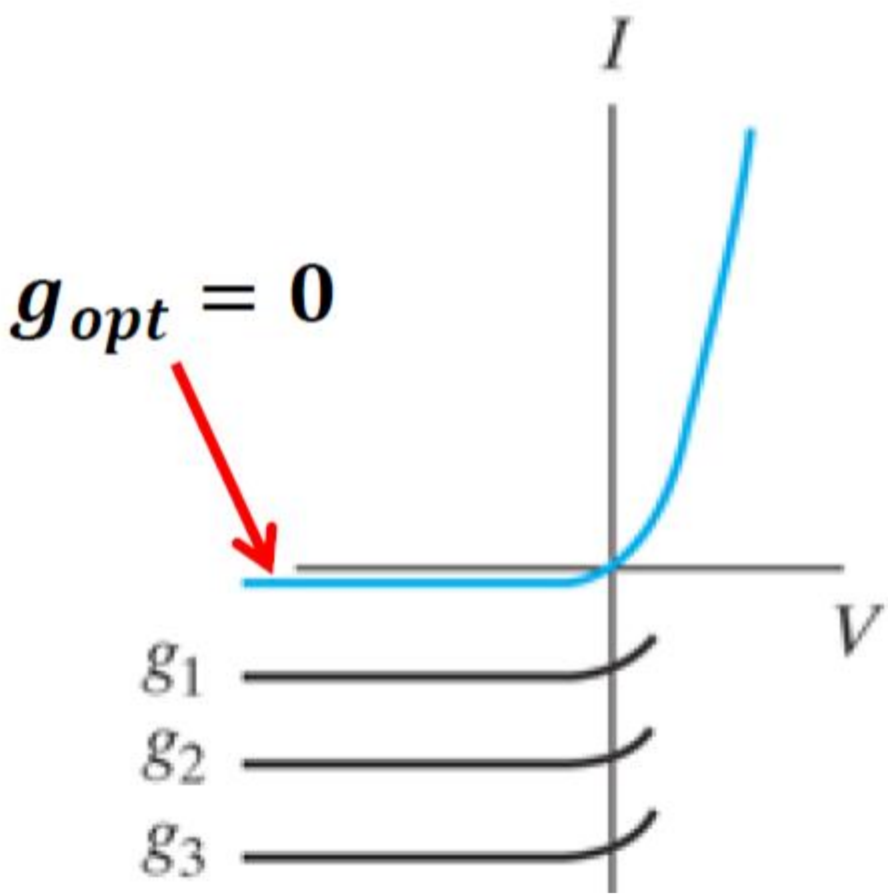
Carriers generated optically in the depletion region are also separated by the junction field.

$$I_{op} = qAg_{op}\left(L_p + L_n + W\right)$$

Optoelectronic devices

Photodiodes

$$I = qA\left(\frac{L_p}{\tau_p}p_n + \frac{L_n}{\tau_n}n_p\right)\left[\exp\left(\frac{qV}{k_BT}\right) - 1\right] - qAg_{op}(L_p + L_n + W)$$

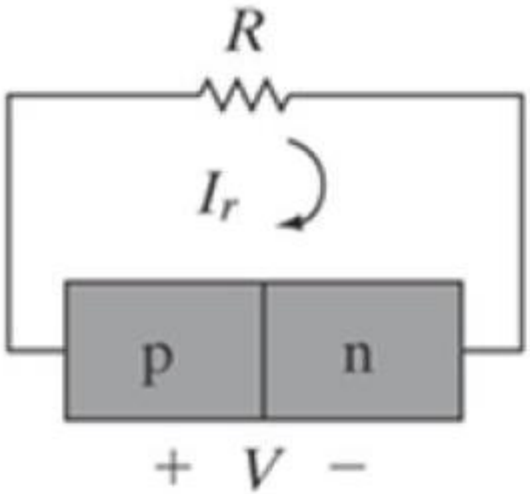


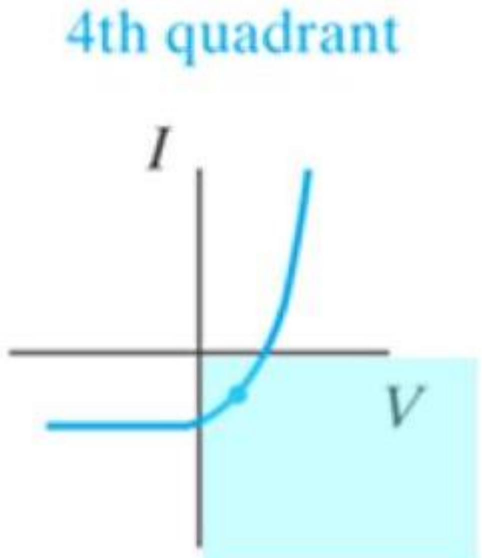
$g_3 > g_2 > g_1$

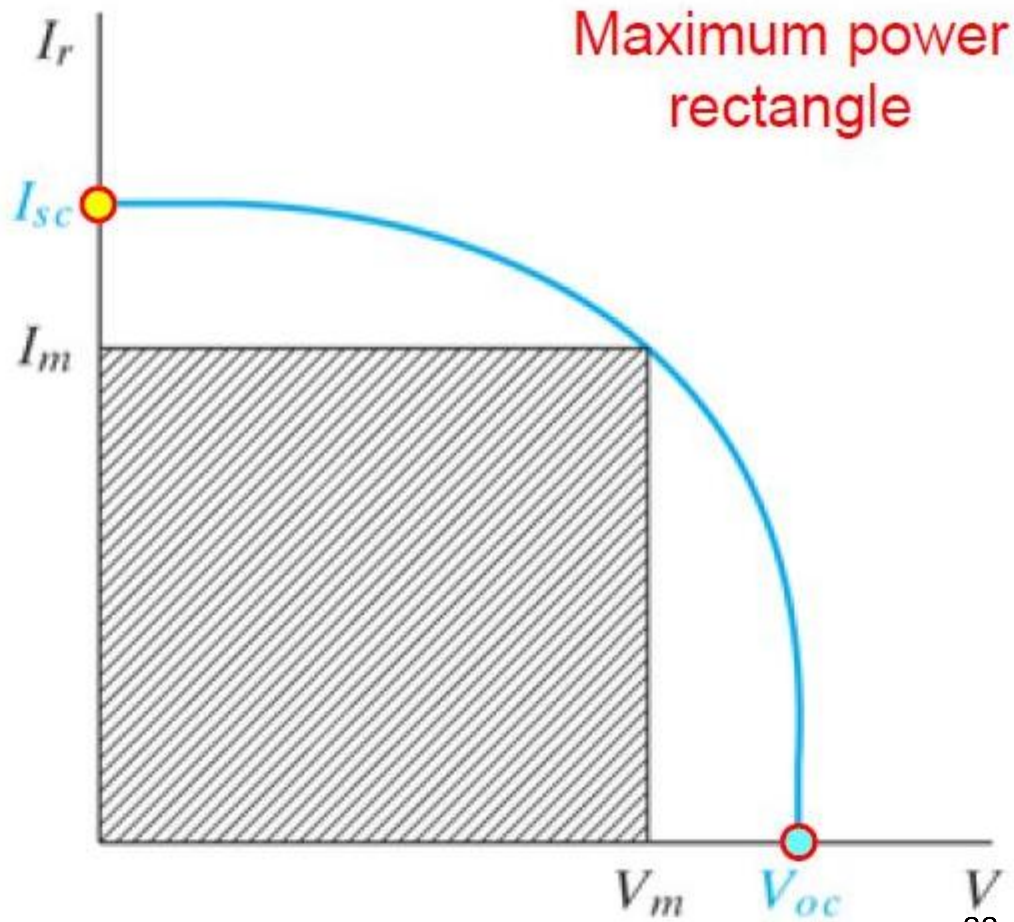
optical generation lowers the ***I-V*** curve

Optoelectronic devices

Solar cells



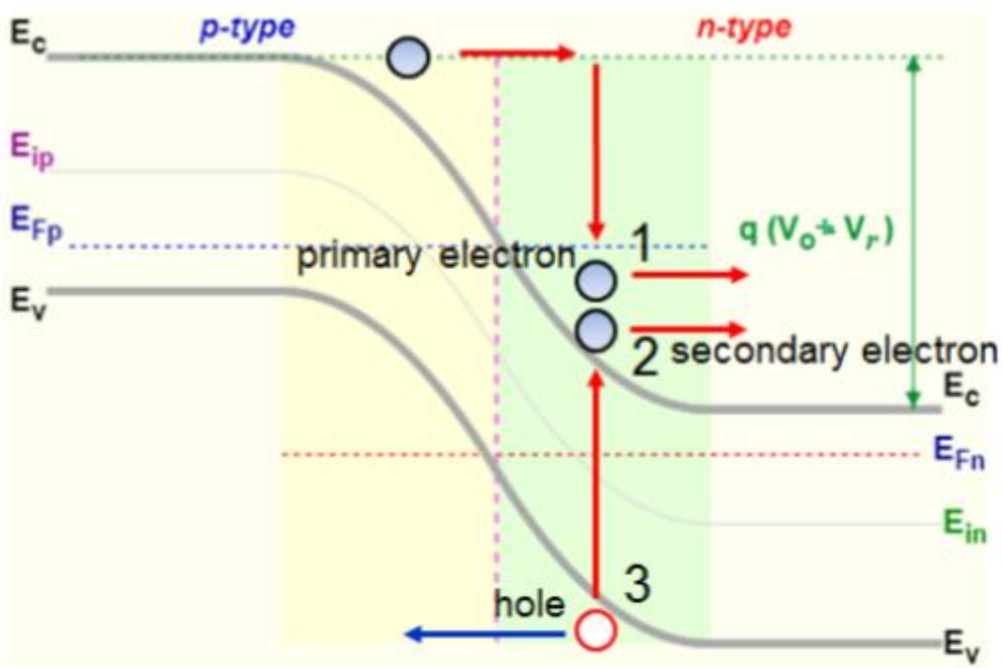




Optoelectronic devices

Avalanche photodiodes

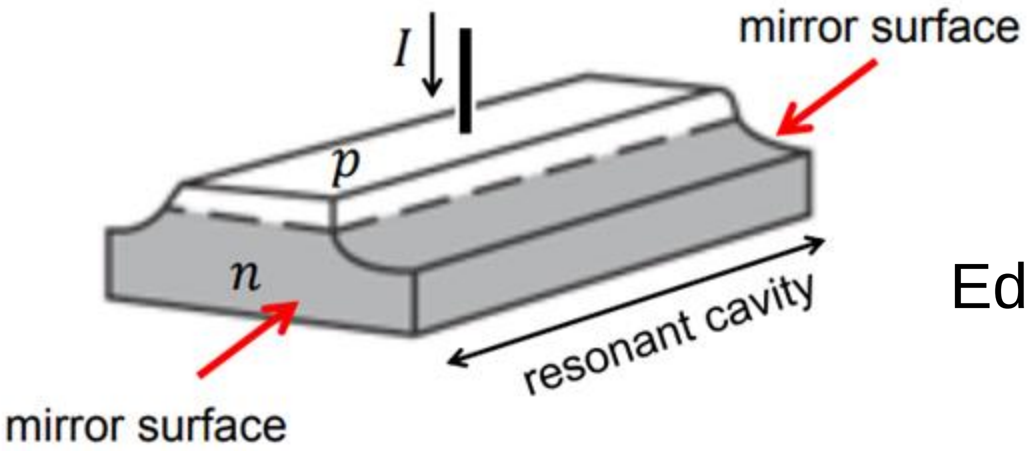
Each photogenerated carrier has the chance to generate EHP by impact ionization. By avalanche multiplication, the signal is essentially amplified.



Optoelectronic devices

Semiconductor lasers

- **Simple *p-n* junction (e.g., GaAs)**



Edge emitting laser

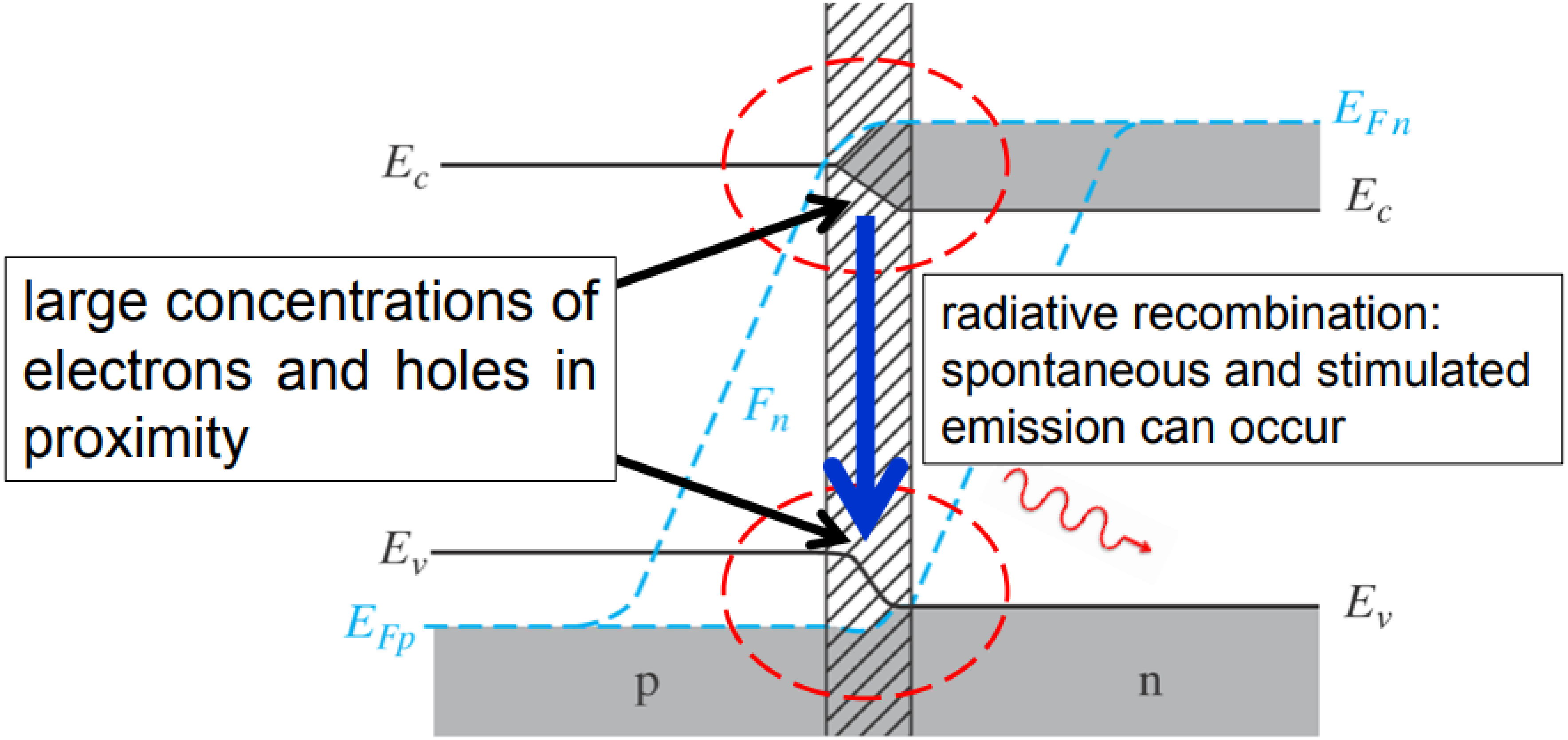
Two ingredients are needed to make a laser:

- population inversion (stable population of excited states)
- resonant cavity to build up a coherent photon population for stimulated emission to occur (coherence)

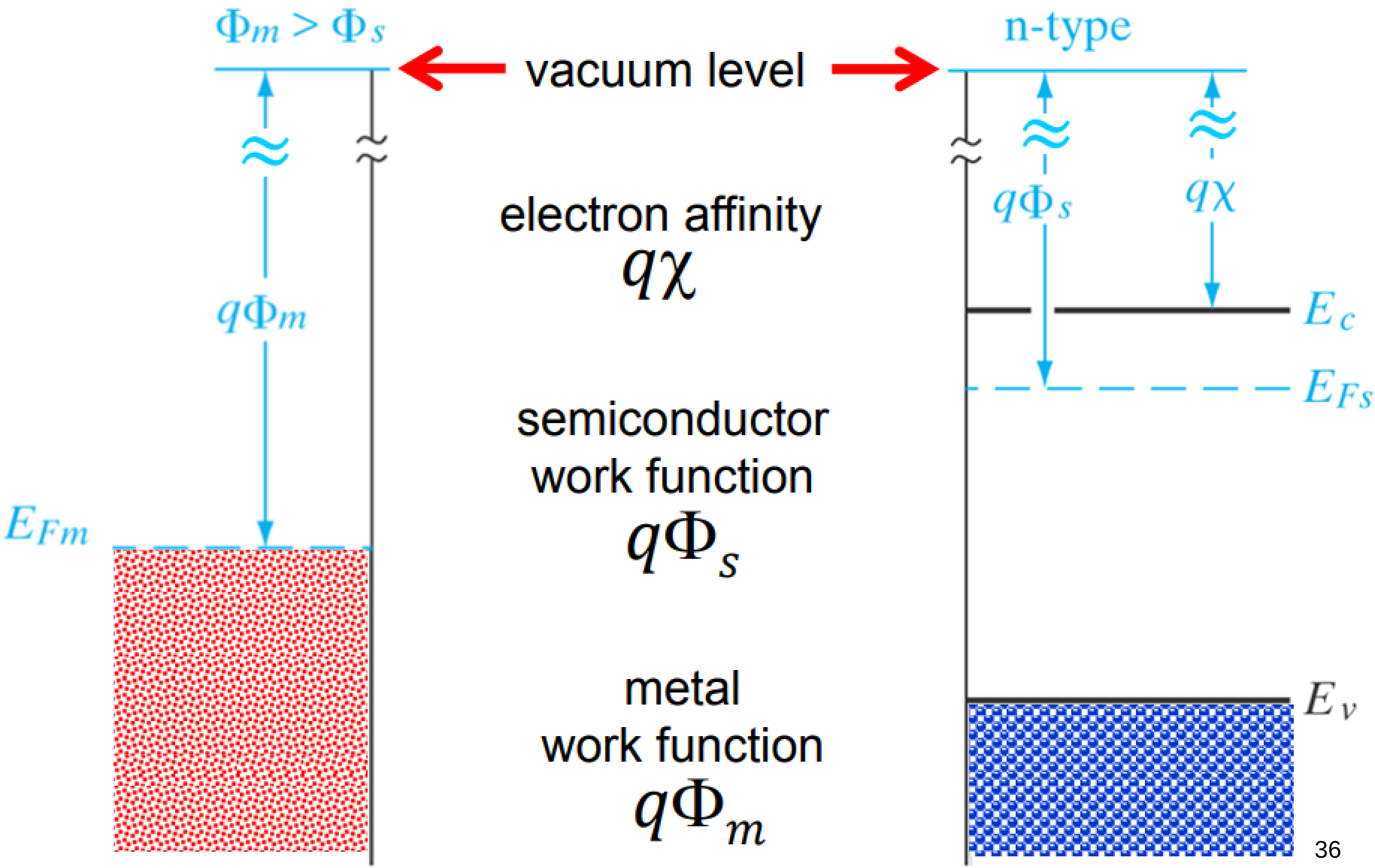
Optoelectronic devices

Semiconductor lasers

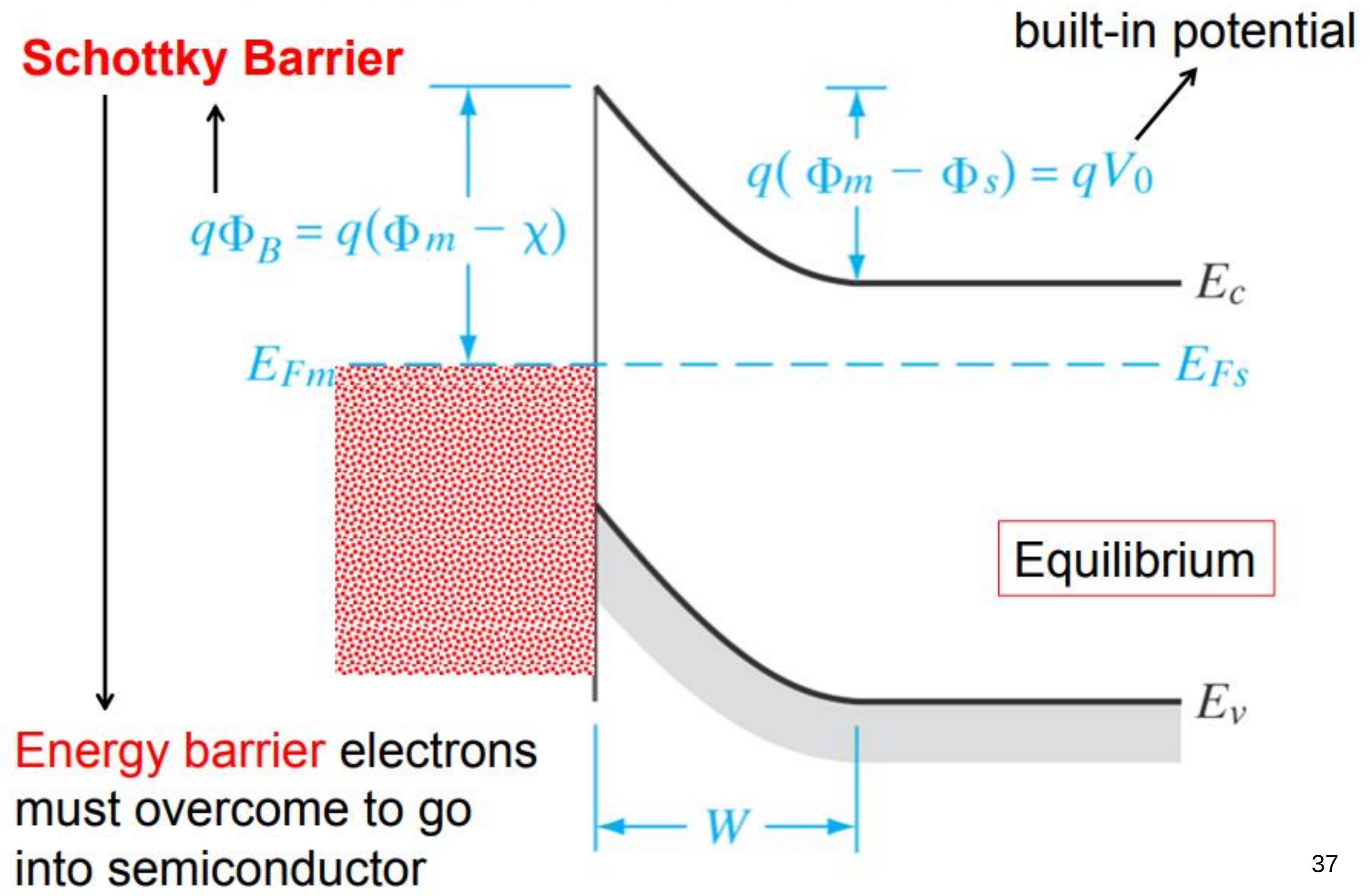
- **Heavily doped *p-n* junction in forward bias**



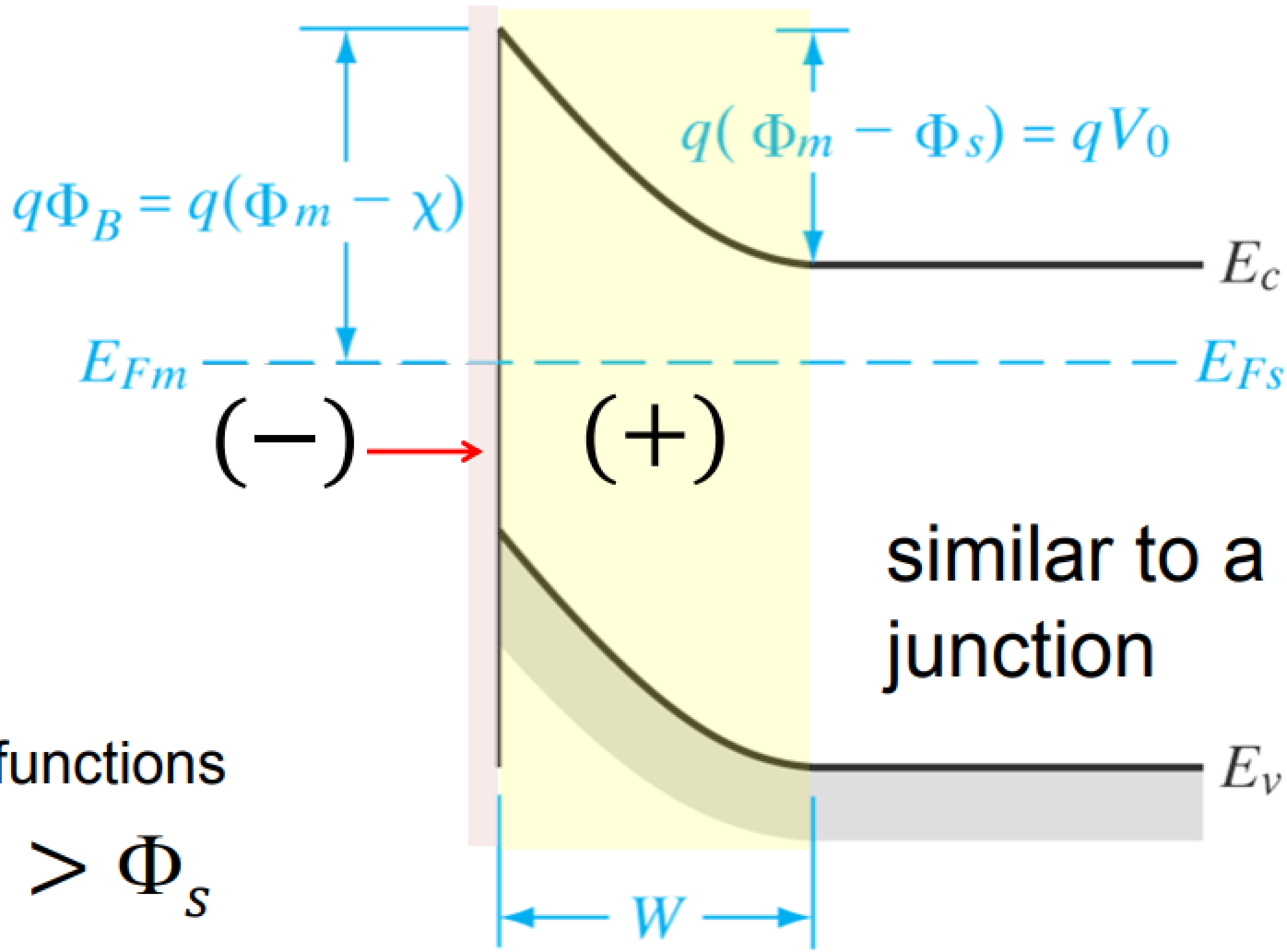
Metal-Semiconductor Junction



Metal-Semiconductor Junction



Metal-Semiconductor Junction



similar to a p^+-n junction

Workfunctions

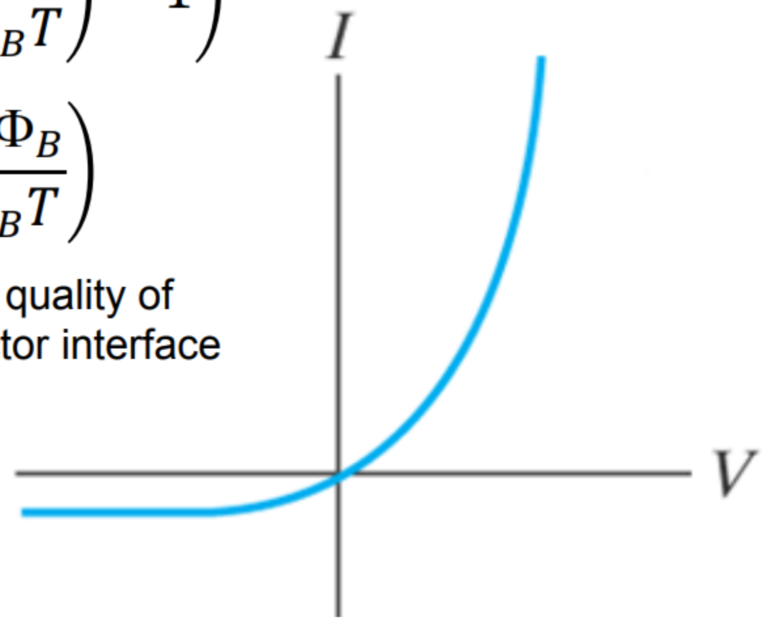
$$\Phi_m > \Phi_s$$

Metal-Semiconductor Junction

$$I = I_0 \left(\exp\left(\frac{qV}{k_B T} \right) - 1 \right)$$

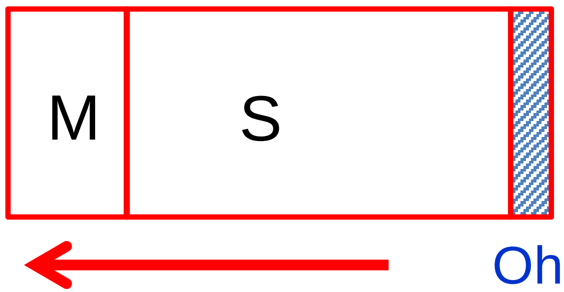
$$I_0 \propto \exp\left(-\frac{q\Phi_B}{k_B T} \right)$$

much depends on quality of metal-semiconductor interface



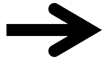
Schottky barrier makes a rectifying junction

Schottky diode



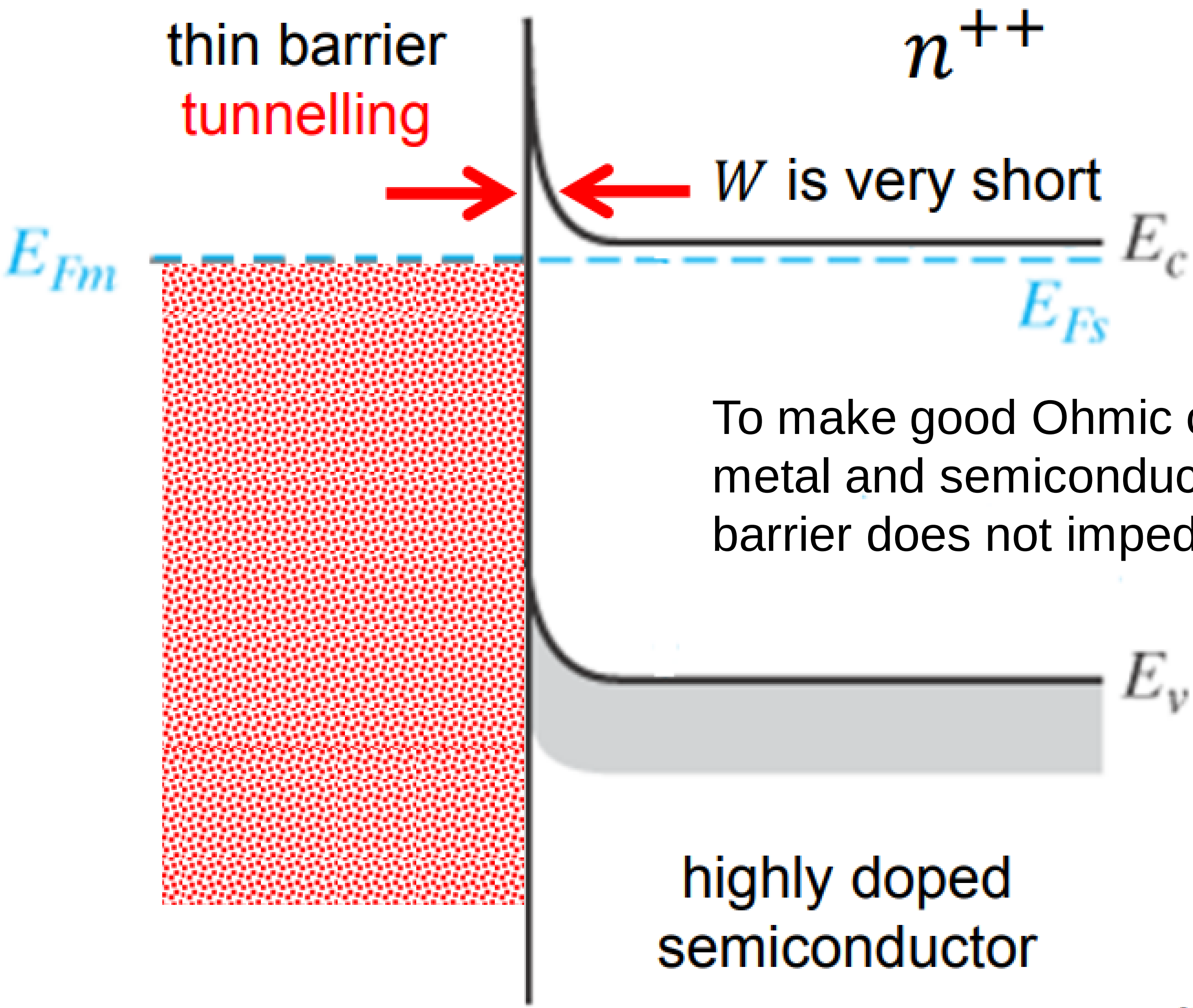
Ohmic contact

Forward bias electron flow by reducing contact potential barrier on semiconductor side



Reverse bias very small electron flow across Schottky barrier on metal side

Metal-Semiconductor Junction



To make good Ohmic contacts between metal and semiconductor so that Schottky barrier does not impede current

27

Ohmic contact (n-type semiconductor)

→ $\Phi_m < \Phi_s$